**ABSTRACT**

This paper employs a bivariate vector autoregressive-generalized autoregressive conditional heteroscedasticity (VAR-GARCH) model recently developed by Ling and McAleer (2003) to examine the impact of oil price fluctuations on stock market returns in the Kingdom of Saudi Arabia over the period from January 1, 2007 to December 31, 2011. The proposed model is estimated using maximum likelihood method under the assumption of multivariate normal distributed error terms. The log likelihood function is maximized using Marquardt’s numerical iterative algorithm to search for optimal parameters. Empirical evidence from daily returns on the Saudi stock market (Tadawul) index and daily crude oil prices suggests that crude oil price fluctuations leads to increase stock market returns volatility during the period of the study.

**Keywords:** Oil price fluctuations; Stock market, Index returns; conditional variance; Saudi Arabia.

**1. INTRODUCTION**

Over the years, crude oil is arguably the most influential physical commodity in the world and frequently considered as an important macroeconomic indicator that influences the stock market, aggregate demand, and real economic growth in both developed and developing countries. Oil price fluctuations might affect the global economy through a variety of channels, including transfer of wealth from oil consumers to oil producers, a rise in the cost of production of goods and services, and impact on inflation, consumer confidence, and financial markets. Early empirical investigations found a significant negative correlation between oil price shocks and aggregate output, and this was used as evidence that oil shocks were responsible for almost all recessions in US after World War II; see for example, Hamilton (1983, 2003) and Mork (1989).

Since then, the identification of the connection between crude oil prices and various financial and macroeconomic variables has been a major concern in theory and practice; for example, Uri (1996) asserted the existence of a Granger-causal relationship between oil shocks and employment. Dogrul (2010) concluded that the real price of oil may improve long-term employment forecasts. Chaudhuri and Daniel (1998) and Askari and Krichene (2010) documented that oil price fluctuations were significantly responsible for exchange rate movements. Cunado and Perez (2005) found that the rise of oil price could cause inflation in some Asian countries. Lardic and Mignon (2008) find evidence for asymmetric cointegration between oil prices and GDP for the G7 and the European countries. By performing a seven vector auto-regression (VAR) framework to analyze the impact of oil price shocks on the system of equations, Burbridge and Harrison (1984) found that the oil price has a significant influence on the industry production of the US and the UK. Like Hamilton (1983), Mork (1989) also employs a six variable VAR model with quarterly observations. However, he extends the sample period to include the oil price collapse in 1986, to investigate if the strong relationship between oil prices and the GNP holds. His results confirm the same negative correlation between oil price increases and the GNP. Kilian (2009) suggests that oil price shocks can have different effects on the real economy depending on whether oil shocks are attributed to global supply shocks or to global demand shocks (see e.g. Kilian 2008b; and Kilian and Park 2009).

As the price fluctuations of crude oil play an important role on a variety of economic activities, such as aggregate demand, inflation, exchange rate, export, import, employment and real economic growth, it is natural to expect that oil price shocks may well have impacts on the stock market. In recent years, the dynamic relationship between oil prices and stock market has been one of the most studied research subject in financial
market literature. A clear understanding of this relationship is of crucial importance for various professional and market participants, such as financial hedger, portfolio manager, asset allocators, or other financial analysts. It has become widely accepted that changes in the price of crude oil are an important factor for driving fluctuations in stock prices especially after the major oil price shocks of the 1970s. The extent to which stock markets are affected by oil prices can be explained by referring to the theory of equity valuation where stock price is obtained by simply discounting all expected future cash-flows at the investors’, required rate of return. Since corporate cash-flows and discount rate reflect economic conditions (inflation, interest rates, production costs, income, economic growth, and market confidence, etc.) which can be influenced by oil shocks, stock prices may react significantly to patterns in oil price fluctuations (Aouri, et al., 2011).

The remainder of this paper is organized as follows. Section 2 provides literature review on the topic and also provides a simple theoretical model on the link between oil prices and stock market. Section 3 introduces the econometric methodology employed and in Section 4 the data and their statistical properties are reported. Section 5 discusses the empirical results. Finally, Section 6 concludes the paper.

2. LITERATURE REVIEW AND THEORETICAL BACKGROUND ON THE LINK BETWEEN OIL PRICES AND STOCK MARKET

2.1 Literature Review

Following the major crude oil price shocks of the 1970s, a growing body of published researches has emerged on the link between energy prices and stock prices. Most of the research has focused on the developed countries and the majority of these researches have shown significant effects of crude oil price fluctuations on stock market returns. Jones and Kaul (1996) investigated the stock markets of the United States, Japan, Canada and the United Kingdom and their reaction to oil price shocks on the basis of the standard cash flow dividend valuation model. Based on quarterly data they found that for the US and Canada this reaction can be accounted for entirely by the impact of the oil shocks on cash flows. The results for Japan and the UK were nevertheless inconclusive. Huang et al. (1996) applied unrestricted vector autoregressive (VAR) which confirmed a significant relationship between some US oil company stock returns and oil price changes. Conversely, they found no evidence of a relationship between oil prices and market indices such as the S&P500. Sadorsky (1999) applied an unrestricted VAR with GARCH effects to US monthly data and identified that oil price shocks, and its volatility played an important part in explaining US real stock returns and the movement of oil price explained more than interest rates for the forecasting variances. By conducting a Granger causality test within the context of a VAR model, Ciner (2001) concluded that a statistically significant relationship existed between real stock returns and oil price futures, but that the connection was non-linear. El-Sharif et al. (2005) examined the links between oil price movements and stock returns in the UK oil and gas sector. They found a strong interrelationship between the two variables. More recently, Nandha and Faff (2008) investigated global equity indices with 35 industrial sectors and show that oil price rises have a negative impact on stock returns for all sectors except mining, and oil and gas industries. Based on the analysis to the US and 13 European countries and use of a multivariate VAR analysis, Park and Ratti (2008) concluded that the impact of oil price shocks on oil-importing countries’ stock market is negative while the impact on oil-exporting countries’ stock market is positive. Malik and Hammoudeh (2007) uses an asymmetric version of the BEKK–GARCH(1,1) model look at the volatility transmission among the US equity markets, the global crude oil market, and three Gulf equity markets including Bahrain, Kuwait, and Saudi Arabia. They show that Gulf equity markets receive volatility from the oil market, but stock market volatility only spills over into the oil market in the case of Saudi Arabia. Hammoudeh and Choi (2007) and Nandha and Hammoudeh (2007) further document that oil plays an important role in emerging stock markets, they revealed that the relationship between crude oil shocks and stock markets seems to be significantly evident and positive. In contrast, Apergis and Miller (2009) have examined whether structural oil-market shocks affect stock prices in eight developed countries. They found that international stock market returns do not respond significantly to oil price shocks. At the industry level, Kilian and Park (2009) find evidence that the impact of oil price shocks on equity returns varies significantly from one industry to another, depending on the underlying causes of the oil price shocks. In recent attempts, Aouri and Nguyen (2010) shift attention to short-term links between oil price fluctuations and stock prices in the aggregate as well as sector-by-sector in Europe. Based on various econometric techniques, they suggested that the sensitivity of sector stock returns to oil price changes differs greatly from one sector of activity to another. More interestingly, their out-of-sample analysis shows that there are substantial diversification benefits to adding the oil asset to a diversified
portfolio of stocks, as doing so significantly improves the portfolio’s risk return characteristics. Finally, Ono (2011) utilizes a VAR model to examine the impact of oil prices on real stock returns for Brazil, Russia, India and China. He tests the responses to linear, non-linear and asymmetric oil price shocks. The results suggest that the real stock returns of China, India and Russia responded statistically significant positively to some of the oil price indicators. Furthermore, asymmetric effect found statistically significant for India. Although most of these studies confirmed the impact of oil price shocks on stock market returns in some developed countries, the results from such studies cannot be generalized to other countries. Consequently, this paper extends the understanding on the dynamic relationship between oil price fluctuations and stock market returns by using data from the Kingdom of Saudi Arabia, which helps to fill in the gap. The paper employs a multivariate vector autoregressive-generalized autoregressive conditional heteroscedasticity (VAR-GARCH) model recently developed by Ling and McAleer (2003) to simultaneously estimate the mean and conditional variance of oil and stock market index returns, thus avoiding the generated regressor problem associated with the two-step estimation process found in many earlier studies (Pagan, 1984).

2.2 On the theoretical link between oil price and stock return

Based on a simple illustration, Huang et al. (1996) suggest that macroeconomic variables such as commodity price can have a significant impact on the stock return of a firm. Assuming firm \( i \) generates an infinite stream of constant expected cash flow, then the stock price of that firm, \( \hat{p}(p_i) \), is simply the present value of expected future cash flow, \( E(CF) \), discounted by discount rate, \( r \), or more formally:

\[
P_i = \frac{E(CF)}{E(r)}.
\]

Where \( E(\hat{\cdot}) \) is the expectation operator, and it follows that the realized stock returns can be expressed approximately as follows:

\[
R_i = \frac{dp}{p} = \frac{d(E(CF))}{E(CF)} - \frac{d(E(r))}{E(r)}
\]

Here, \( R_i \) is a stock return, computed as \( \log\left(\frac{SP_t}{SP_{t-1}}\right)*100 \); and \( d(\hat{\cdot}) \) is the differentiation operator. We notice that stock returns are influenced by systematic movements in expected cash flows as well as by discount rates.

There are two channels through which oil prices can impact stock prices (returns). First, oil is considered an input in the production process. A rise in the oil price raises the cost of production, which will depress aggregate stock prices. Second, as lucidly explained by Huang et al. (1996), expected oil prices also affect stock returns via the discount rate, which consists of both the expected inflation rate and the expected real interest rate. Since, both expected inflation and interest rates are influenced by oil prices, for a net importer of oil an oil price increase will put downward pressure on the country’s foreign exchange rate and upward pressure on the expected domestic inflation rate. A higher expected inflation rate raises the discount rate, which has a negative effect on stock returns.

3. ECONOMETRIC METHODOLOGY

For a wide range of financial data series, time-varying conditional variances can be well explained through the use of the Autoregressive Conditional Heteroscedasticity (ARCH) of Engle (1982) and the Generalized ARCH (GARCH) developed by Bollerslev (1986) see for example (Agnolucci, 2009; Hassan and Malik, 2007; and Kang et al., 2009). However, it is commonly accepted that multivariate GARCH specifications such as BEKK

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1. This finding is consistent with those of several other papers, for which using futures contracts on traded commodities as part of existing portfolios of stocks improves overall returns (Satyanarayan and Varangis, 1996; Geman and Kharoubi, 2008).
2. For more information about Saudi stock market (Tadawul) see for example Talat et al., 2011.
3. It should be noted that not all models in the literature have the generated regressor problem. For example, Schwert (1989) suffers from this problem while other studies like Kearney (2000) do not.
4. A time series is said to be heteroscedastic if its variance changes over time, otherwise it is called homoscedastic.
model of Engle and Kroner (1995), CCC-GARCH model of Bollerslev (1990) or DCC-GARCH model of Engle (2002) are more relevant than univariate settings as far we are concerned by the volatility transmissions issue among multiple financial variables. To investigate the impact of crude oil price fluctuations on stock market returns in Saudi Arabia, this paper use the VAR(1)-GARCH(1,1) model proposed by Ling and McALeer (2003). Apart from the fact that this model performs quite well in empirical modeling of volatility spillovers as noted by the aforementioned studies, one should remark its two major advantages. First, it permits a multivariate analysis of conditional volatility of the studied series as well as of conditional cross effects and volatility spillovers between series. Second, it provides efficient estimates of the model's unknown parameters while avoiding the computational complications of some multivariate volatility models, such as the full-parameterized BEKK-GARCH, when the number of variables we consider becomes significant. This approach was applied by, among others, Chan et al., (2005), Chang et al., (2011) and Hammoudeh et al., (2009) to various economic issues and appears to provide meaningful and interpretable coefficients.

The VAR(1)-GARCH(1,1) considered in this paper has the following specification for the conditional mean:

\[
\begin{align*}
R_t &= \mu + \Phi R_{t-1} + \varepsilon_t, \\
\varepsilon_t &= H_t^{1/2} \eta_t,
\end{align*}
\]

where

- \( R_t = (r^s_t, r^o_t)' \) is the vector of returns on the stock market index and oil price index respectively.
- \( \Phi \) refers to a (2×2) matrix of coefficients of the form:
\[
\Phi = \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix}
\]
- \( \varepsilon_t = (\varepsilon^s_t, \varepsilon^o_t)' \) is the vector of the error terms of the conditional mean equations for stock and oil returns respectively.
- \( \eta_t = (\eta^s_t, \eta^o_t)' \) refers to a sequence of independently and identically distributed (i.i.d) random errors;
- \( H_t = \begin{pmatrix} h^s_t & h^{so}_t \\ h^{os}_t & h^o_t \end{pmatrix} \) is the matrix of conditional variances of stock market and oil returns with \( h^s_t \) and \( h^o_t \) being the conditional variances of \( r^s_t \) and \( r^o_t \) respectively. Their time series dynamics are modelled as follows:

\[
\begin{align*}
h^s_t &= c^s_t + \beta^s_{11} h^s_{t-1} + \alpha^s_{11} (\varepsilon^s_{t-1})^2 + \beta^s_{21} h^{os}_{t-1} + \alpha^s_{21} (\varepsilon^o_{t-1})^2 \\
h^o_t &= c^o_t + \beta^o_{11} h^o_{t-1} + \alpha^o_{11} (\varepsilon^s_{t-1})^2 + \beta^o_{21} h^{os}_{t-1} + \alpha^o_{21} (\varepsilon^o_{t-1})^2
\end{align*}
\]

Obviously, Eqs. (2) and (3) assume that negative and positive shocks of equal magnitude have identical effects on conditional variances. The Eqs. also show how volatility is transmitted over time and across the two markets under investigation. The cross values of error terms, \( (\varepsilon^s_{t-1})^2 \) and \( (\varepsilon^o_{t-1})^2 \), represent the return innovations in the oil market and to the corresponding stock market at time (t-1), and thus capture the impact of direct effects of shock transmission, as well as those of lagged conditional volatilities, \( h^s_{t-1} \) and \( h^o_{t-1} \), which directly accounts for the transfer of risk between markets. Under some regularity conditions, the roots of the equation \( \begin{vmatrix} I_2 - A L - B L \end{vmatrix} = 0 \) must be outside the unit circle in order to guarantee stationarity, and the expression \( (I_2 - AL) \) and \( BL \) satisfy some other identifiability conditions as proposed by Jeantheau (1998), where \( L \) is a lag polynomial, \( I_2 \) is a (2×2) identity matrix, and \( A \) and \( B \) are defined as:

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5 In general terms, volatility refers to the fluctuations observed in some phenomenon over time. In terms of modelling and forecasting literature, it means “the conditional variance of the underlying asset return” (Tsay, 2010).

6 The optimal number of lags for the VAR system was chosen on the basis of commonly used information criterion.
Let $\rho$ be the constant conditional correlation (CCC); the conditional covariance between oil returns and stock market returns is modelled as:

$$h_{it}^{oo} = \rho \sqrt{h_{it}^A} \sqrt{h_{it}^B}$$

As specified previously, the empirical model simultaneously allows long-run volatility persistence as well as shock and volatility transmissions between the oil and stock markets under consideration. Note that the CCC assumption can be viewed as restrictive given that correlation coefficient is likely to vary over time according to changes in economic and market conditions. Nevertheless, the statistical properties of a VAR-GARCH model accounting for dynamic conditional correlation have not yet been analyzed theoretically (Chang et al., 2011; McAleer et al., 2009). Following Ling and McAleer (2003), the quasi-maximum likelihood estimation (QMLE) method of Bollerslev and Wooldridge (1992) is used to estimate the empirical model in order to take into account the fact that normality condition is often rejected for majority of macroeconomic and financial series.

4. DATA AND PRELIMINARY ANALYSIS

The main purpose of this paper is to investigate the impact of crude oil price fluctuations on the stock market returns for the Kingdom of Saudi Arabia; one of the largest producing crude oil in the world. According to Park and Ratti (2008)’s research, the stock market’s response to oil price fluctuations partly depended on whether the country was oil importing or oil exporting. The data set used in this study consists of a daily oil price time series and a daily general market index, TASI, time series. Both series span from January 1, 2007 to December 31, 2011. Daily closing prices of TASI have been taken from the Saudi Stock Market website (http://www.tadawul.com.sa). While world oil price data are extracted from the Energy Information Administration (EIA). The Brent spot prices are used to represent the international crude-oil market since they usually serve as reference prices for pricing crude oil and many other derivatives products using oil as underlying asset. Unlike the majority of previous studies which employ low frequency data (yearly, quarterly, monthly, and weekly), this paper uses daily data in order to adequately capture the rapidity and intensity of the dynamic interactions between oil and stock prices. Daily returns are calculated from daily price data by taking the natural logarithm of the ratio of two successive prices.

For both series, daily returns, in percentage, are defined as:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) * 100$$

where $r_t$ is the return of the index at time $t$. $P_t$ and $P_{t-1}$ are the closing market index of TASI at the current day and previous day, respectively. Table 1 summarizes the returns of crude oil price changes along with the stock market returns of the TAS general Index statistics.

(Table 1 about here)

The results in Table 1 show that the crude oil market experienced higher returns than the general market index over the sample period. In view of the maximum and minimum values of both series it can be noted that fluctuations in oil prices have been greater than of Saudi stock market returns over the period of study. In the same way, the value of standard deviation (an indication of unconditional variance in return series) regarding the value of mean is another proof for high volatility and risky nature of oil market in comparison with the stock market. The results also indicate that both return series do not conform to normal distribution but display negative skewness (the distribution has a long left tail) for returns of stock market index and positive skewness (the distribution has a long right tail) for the oil price returns, in addition to that, a highly leptokurtic distribution is also observed for both returns series. The Ljung–Box statistics for serial auto-correlations on return and squared return series indicate that the null hypothesis of no auto-correlations up to the 12th order is rejected at almost 1% significant level and confirms the existence of auto-correlations in the two return series. Finally, the results shows strong evidence of ARCH effects for both series considered at 1, 6, and 12 lags which thus supports the decision to employ a GARCH modeling approach to examining volatility transmission between series.
crude oil price fluctuations and stock market returns. Fig. 1 (Fig. 2) provides an illustration of the prices (returns) in the TASI, and for crude oil price changes.

(Table 1 and 2 about here)

According to Fig. 2, volatility clustering is apparent, that is, high volatility in one period tends to be followed by high volatility in the next period, indicating the plausibility of the GARCH structure.

5. EMPIRICAL RESULTS

As reported in the data description part when the residuals were examined for heteroscedasticity, ARCH-LM test provided strong evidence of ARCH effects in the residual series for both oil prices and stock market returns, which indicates that it is possible now to proceed with modelling of the returns volatility by using GARCH framework. The proposed model is estimated using maximum likelihood method under the assumption of multivariate normal distributed error terms. The log likelihood function is maximized using Marquardt’s numerical iterative algorithm to search for optimal parameters. Beside the estimation output of the VAR(1)-GARCH(1,1) model, diagnostics test results are also provided to see whether there still ARCH effects left in the estimated model. Table 2 shows the parameter estimates of the model.

(Table 2 about here)

Estimation results of the return and volatility dependencies by applying VAR(1)-GARCH(1,1) model are reported in Tables 2 for the stock and oil markets. Regarding the interdependence of returns in mean equations, the results find that a one-period lagged oil returns parameter significantly affect their current values and also the returns of Saudi stock market which imply that past oil returns can better be used to predict future of the two series considered.

As for the estimates of ARCH and GARCH coefficients in the conditional variance equations, they are statistically significant for both series. The significant of past own conditional volatility (GARCH-term) suggests that past values of the conditional volatility of stock market returns can be employed to forecast future volatility. The results also show that the current conditional volatility of Saudi stock market depends on past shocks affecting return dynamics since ARCH-terms are highly significant. A closer inspection of the above coefficients reveals that in general; conditional volatility does not change very rapidly as the ARCH-terms measuring the impact of past shocks on conditional volatility are relatively small in size. On the other hand, the GARCH-terms, which capture the impact of past volatility on current volatility, are substantially large, indicating gradual fluctuations over time.

The empirical findings regarding the volatility transmission between oil and stock markets in Table 2 indicate that the conditional volatility of the stock market returns is affected by innovations in the oil market as indicated by the significance of the coefficient on \( \hat{e}_{t-1}^o \). Apparently, a shock originating from the oil market leads to increase stock market returns volatility. In addition, there is strong evidence to suggest that past volatility of the oil market is transmitted to stock market because the coefficient associated with \( h_{t-1}^o \) is significant. On the other hand, the statistical significance of the coefficients of \( \hat{e}_{t-1}^s \) and \( h_{t-1}^s \) in the conditional volatility equation for oil returns suggests that stock market volatility behaves dependently on the changes (shock and volatility) occurred in the oil market.

The residual diagnostic tests are reported in the last part of Table 2. The Ljung–Box Q-statistic suggests that the null hypothesis of no autocorrelation cannot be rejected; thus, the residuals are free of autocorrelation. The ARCH-LM test suggests that the null hypothesis of no ARCH effects cannot be rejected; implying that the residuals do not suffer from the ARCH effects which imply that the conditional variance equation is well specified.

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8 This means large changes tend to be followed by large changes and small changes tend to be followed by small changes (Mandelbrot, 1963).
9 See an appendix.
10 If the variance equation of GARCH model is correctly specified, there should be no ARCH effect left in the residuals.
6. CONCLUSION

The sharp and persistent increases in oil prices during the last few decades have attracted attention from policy makers as well as macroeconomists, and led to much research concerning the impacts of oil price shocks on the economy as a whole. In particular, recent studies have documented empirically that high oil prices can have significant impact on stock market returns. In the same track, this paper tried to look at the impact of crude oil price fluctuations on stock market returns. Based on daily observations of Tadawul all share index (TASI) and crude oil prices over the period 1st January 2007 to 31st December 2011, the paper employed a bivariate vector autoregressive-generalized autoregressive conditional heteroscedasticity (VAR-GARCH) model recently developed by Ling and McAleer (2003) to simultaneously estimate the conditional mean and conditional variance of oil and stock market returns. Empirical results of the paper indicate that an increase in oil prices leads to increase stock market returns volatility. In addition, there is evidence to suggest that past volatility of oil market is transmitted to stock market in the Kingdom of Saudi Arabia during the period of the study.

REFERENCES

Table 1. Summary statistics for stock market returns and oil price changes

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Stock Returns</th>
<th>Oil Price Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (%)</td>
<td>-0.019929</td>
<td>0.044536</td>
</tr>
<tr>
<td>Minimum</td>
<td>-10.32845</td>
<td>-13.06537</td>
</tr>
<tr>
<td>Maximum</td>
<td>9.087370</td>
<td>16.40973</td>
</tr>
<tr>
<td>Std. dev. (%)</td>
<td>1.679530</td>
<td>2.765960</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.797523</td>
<td>0.080885</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>10.53532</td>
<td>7.284939</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>3223.334***</td>
<td>999.0199***</td>
</tr>
<tr>
<td>ARCH(1)</td>
<td>41.04242***</td>
<td>65.23083***</td>
</tr>
<tr>
<td>ARCH(6)</td>
<td>276.5523***</td>
<td>242.7017***</td>
</tr>
<tr>
<td>ARCH(12)</td>
<td>296.3569***</td>
<td>266.7683***</td>
</tr>
<tr>
<td>Ljung-Box (6)</td>
<td>14.4301***</td>
<td>17.540***</td>
</tr>
<tr>
<td>Ljung-Box (12)</td>
<td>19.671**</td>
<td>27.299***</td>
</tr>
<tr>
<td>Ljung-Box (6) on squared returns</td>
<td>458.53***</td>
<td>533.72***</td>
</tr>
<tr>
<td>Ljung-Box (12) on squared returns</td>
<td>708.58***</td>
<td>913.42***</td>
</tr>
</tbody>
</table>

*, ** and *** indicate significant at the 10%, 5% and 1% levels respectively.

Table 2. Estimation results of VAR(1)-GARCH(1,1) model for oil and stock market returns

<table>
<thead>
<tr>
<th>Variables</th>
<th>Stock Market Returns</th>
<th>Oil Price Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional mean equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.028452</td>
<td>0.121849**</td>
</tr>
<tr>
<td>Oil(-1)</td>
<td>0.033301**</td>
<td>0.033282*</td>
</tr>
<tr>
<td>Conditional variance equation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.065344*</td>
<td>0.103036**</td>
</tr>
<tr>
<td>$\left(e_{t-1}^s\right)^2$</td>
<td>0.103868***</td>
<td>0.062682***</td>
</tr>
<tr>
<td>$\left(e_{t-1}^o\right)^2$</td>
<td>0.062682***</td>
<td>0.103867***</td>
</tr>
<tr>
<td>$h_{t-1}^s$</td>
<td>0.869451***</td>
<td>0.919784***</td>
</tr>
<tr>
<td>$h_{t-1}^o$</td>
<td>0.919783***</td>
<td>0.869450***</td>
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<tr>
<td>Diagnostics</td>
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<tr>
<td>LB²(12)</td>
<td>6.5324</td>
<td>7.6921</td>
</tr>
<tr>
<td>ARCH(12)</td>
<td>6.443766</td>
<td>7.297204</td>
</tr>
</tbody>
</table>

LB²(12) and ARCH(12) refer to the empirical statistics of the Ljung-Box test for autocorrelation of order 12 applied to the standardized residuals and the Engle (1982) test for conditional heteroscedasticity of order 12. *, **, and *** indicate the rejection of the null hypothesis of associated statistical tests at the 10%, 5%, and 1% levels respectively.
Fig. 1. Oil prices and stock market index.

Fig. 2. Returns on oil price and stock market index.
APPENDIX

Testing for Heteroscedasticity (ARCH-LM test)

The ARCH LM test is used for testing the conditional heteroscedasticity in the residual series. Its commonly accepted in volatility modelling that one of the most important issues before applying the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) methodology is to first examine the residuals for evidence of heteroscedasticity. To test for the presence of heteroscedasticity in residuals, the Lagrange Multiplier (LM) test for ARCH effects proposed by Engle (1982) is applied.

In summary, the test procedure is performed by first obtaining the residuals $e_t$ from the ordinary least squares regression of the conditional mean equation which might be an autoregressive (AR) process, moving average (MA) process or a combination of AR and MA processes; i.e., an ARMA process. For example, in ARMA (1,1) process the conditional mean equation will be:

$$r_t = \phi_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1}.$$  \hfill (1)

After obtaining the residuals $e_t$, the next step is to regress the squared residuals on a constant and q lags as in the following equation:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \alpha_2 e_{t-2}^2 + \ldots + \alpha_q e_{t-q}^2 + \nu_t.$$  \hfill (2)

The null hypothesis that there is no conditional heteroscedasticity (ARCH effect) up to order q can be formulated as:

$$H_0 : \alpha_1 = \alpha_2 = \ldots = \alpha_q = 0$$  \hfill (3)

against the alternative:

$$H_1 : \alpha_i > 0$$  \hfill (4)

for at least one $i = 1, 2, \ldots, q$.

The test statistic for the joint significance of the q-lagged squared residuals is the number of observations times the R-squared ($TR^2$) from the regression (2). $TR^2$ is evaluated against the $\chi^2 (q)$ distribution. This is an asymptotically locally most powerful test (Rachev et al., 2007).